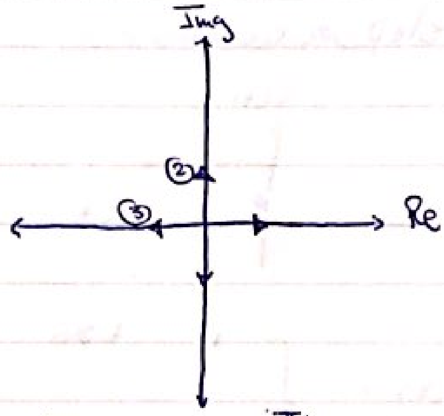
Anas Alloush

Finding t_d Time delay:



$x(t) = \sin(\omega t)$

$x(t - t_d) = \sin[\omega_0 (t - t_d)]$

$= \sin(\omega_0 t - \omega_0 t_d)$

$= \sin(\omega_0 t + \phi_d)$

ϕ_0 = phase delay

$\phi_0 = -\omega_0 t_d \rightarrow t_d = \frac{-\phi_d}{\omega_0}$ always in rad

① $e^{j\pi} = -1$ ② $e^{j\pi/2} = j$

③ $e^{-j\pi} = -1$ ④ $e^{-j\pi/2} = -j$

Addition of Sinusiodals:

$a \cos \underline{\omega_0} t + b \sin \underline{\omega_0} t = \underline{C \cos(\omega_0 t + \theta)}$

$C = \sqrt{a^2 + b^2}$ $\theta = \tan^{-1}\left(\frac{-b}{a}\right)$

Ex. Find. $x(t) = \cos \omega_0 t - \sqrt{3} \sin \omega_0 t$

$a = 1$, $b = -\sqrt{3}$

لازم يكون wo نفس الشي

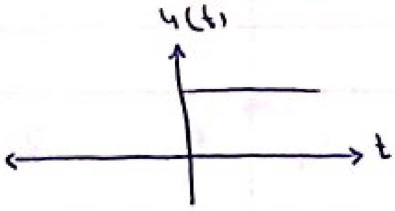
كغير ا لا تتجمع لا تتطرح

Euler's equation:

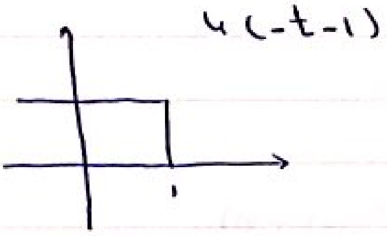
$e^{j\theta} = \cos\theta + j\sin\theta$ $\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$

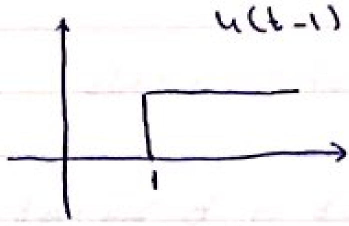
$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$

Unit step ((u(t))):

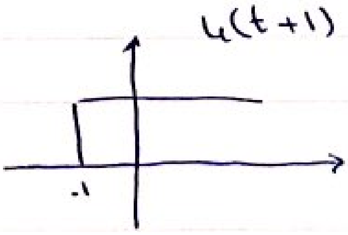


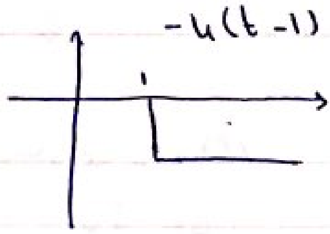
$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{e.w (else where)} \end{cases}$$





$$u(t-1) \begin{cases} 1 & t \geq 1 \\ 0 & t < 1 \end{cases}$$





Re. Time

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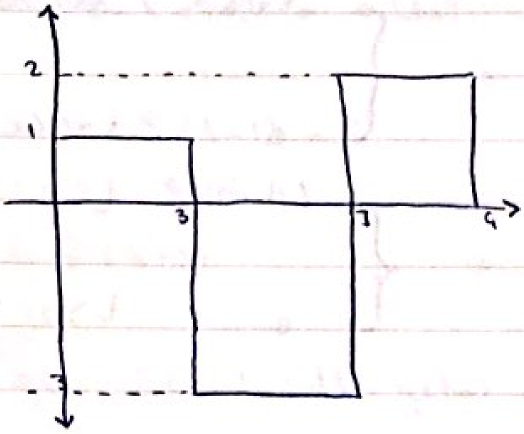
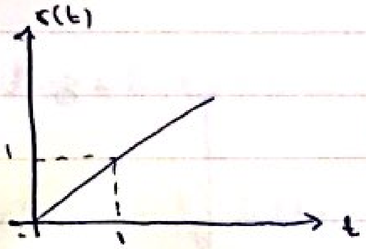
Ex: Sketch $x(t) = u(t) - 4u(t-3) + 5u(t-7) - 2u(t-9)$



Ramp Signal $r(t)$:



Note:

① $r(t) = t \cdot u(t)$

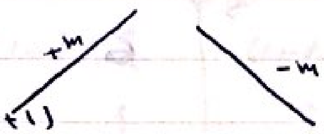
② $A\,r(t) = At \cdot u(t)$

③ $A\,r(at+b) \cdot u(at+b)$

$y = mx + c$

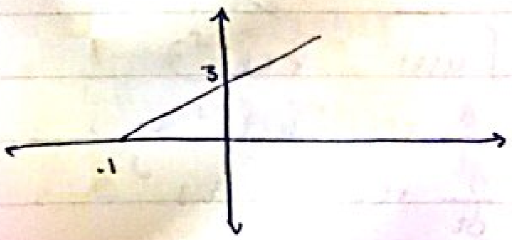
$m = \dfrac{y_2 - y_1}{x_2 - x_1}$

$C =$ الجزء المقطوع من محور الصادات



$$r(t) = \begin{cases} t & t \geq 0 \\ 0 & t < 0 \end{cases}$$

Sketch: ① $x(t) = 3r(t+1) \rightarrow 3(t+1)u(t+1)$



$$= \begin{cases} 3[t+1] & t+1 > 0 \\ 0 & t+1 < 0 \end{cases}$$

$$= \begin{cases} 3t+3 & t > -1 \\ 0 & t < -1 \end{cases}$$

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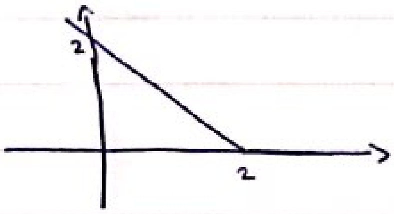
(c) $x(t) = r(-t+2) = [-t+2]\,u(-t+2)$

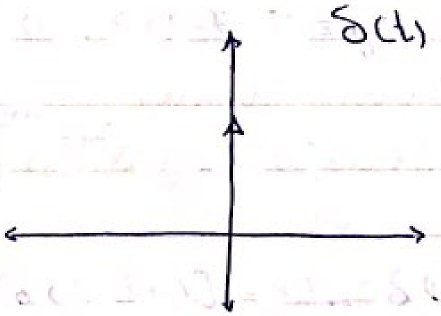


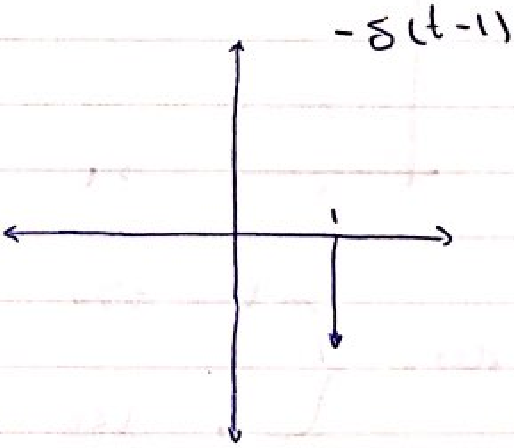
$$= \begin{cases} -t+2 & -t+2>0 \\ 0 & -t+2<0 \end{cases}$$

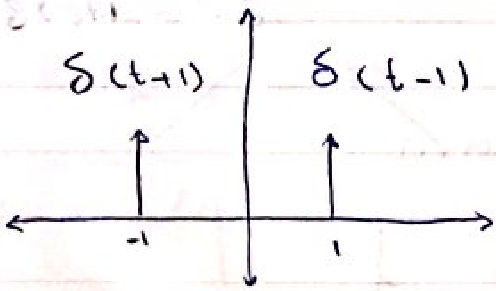
$$= \begin{cases} -t+2 & t<2 \\ 0 & t>2 \end{cases}$$

Impulse function "$\delta(t)$":

$$\delta(t) = \begin{cases} \infty & t=0 \\ 0 & \text{o.w} \end{cases}$$







$$\int_{-\infty}^{\infty} \delta(t) = 1 = u(t)$$

$$\int u(t) = r(t)$$

$$\frac{d}{dt} r(t) = u(t)$$

$$\frac{d}{dt} u(t) = \delta(t)$$

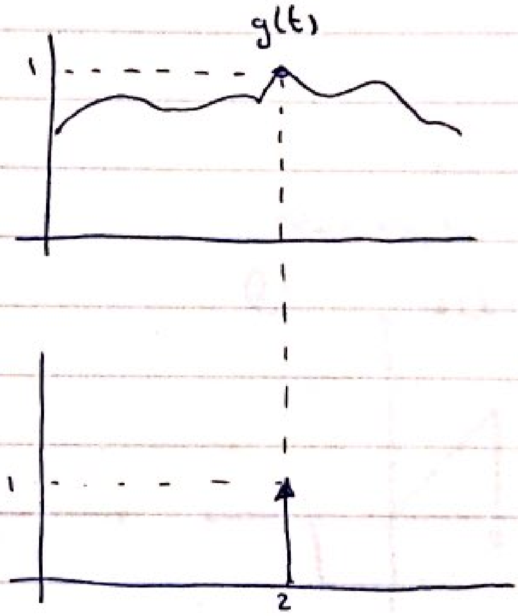
Proposlies:

① $g(t) \cdot \delta(t-t_0) = g(t_0) \cdot A\,\delta(t-t_0)$

Ex: $(t+3)\cdot\delta(t-1) = ?$

$= (1+3)\cdot\delta(t-1)$

$= 4\,\delta(t-1)$



② $\int_{-\infty}^{\infty} g(t)\cdot\delta(t-t_0)\,dt = g(t_0)$

Ex: $\int_{-\infty}^{\infty} e^{-j\omega t}\cdot\delta(t) = ?$

$= 1$

③ $\delta[a(t-t_0)] = \frac{1}{|a|}\,\delta(t-t_0)$

Ex: $\delta(3t) = \frac{1}{3}\,\delta(t)$

$\delta(\frac{t}{2}-1) = \delta[\frac{1}{2}(t-2)]$

$= \frac{1}{1/2}\,\delta(t-2)$

④ $\int_{t_1}^{t_2} f(t)\,\delta(at+b) = \frac{1}{|a|}\cdot f(\frac{-b}{a})$

Ex: $\int_{-\infty}^{\infty} e^{-2(5-t)}\cdot\delta(2-t)$

$a=-1,\ b=2$

$= \frac{1}{|-1|}\,e^{-2[5-(\frac{-2}{-1})]}$

$= 1\cdot e^{-2(5-2)}$

$= e^{-6} = \#$

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Ex: $\int_0^2 (t+1) \cdot \delta(t+1) = 0$

only if the shift is out of the given range

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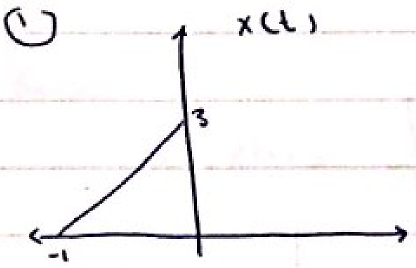
Sketching Signals:

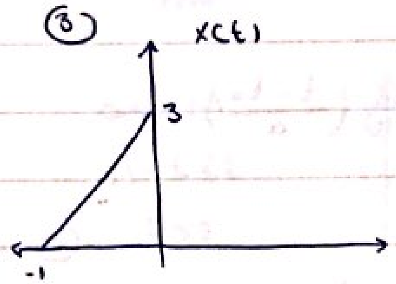
1. Amplitude Scaling
2. Time Scaling
3. Time Shifting

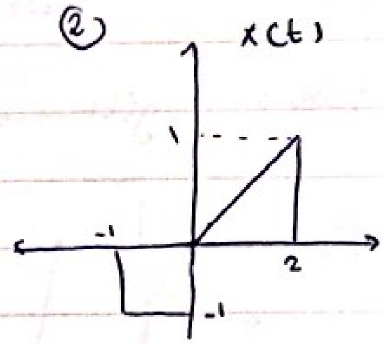
Amplitude Scale

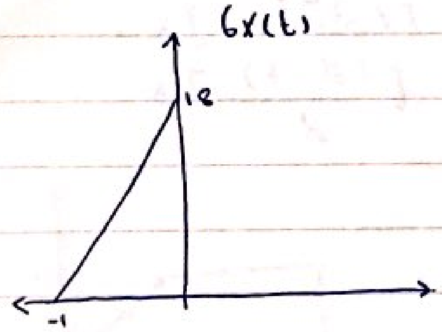
Time Shift

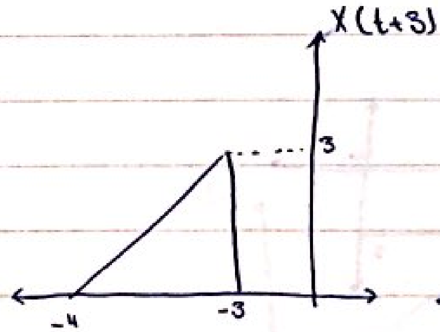
Time Scale

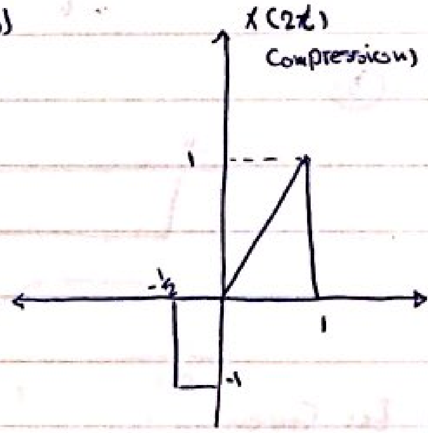


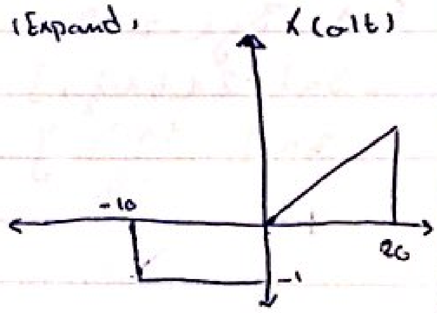




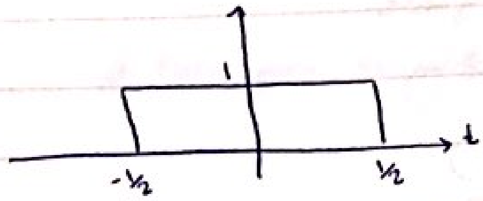








Ex: Given $g(t)$



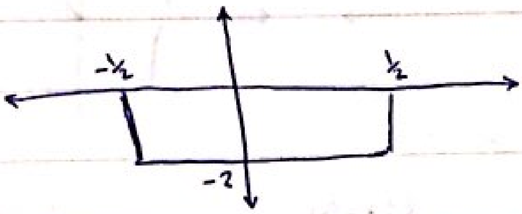
Sketch $-2g\left(\frac{t+2}{4}\right)$

Solution:

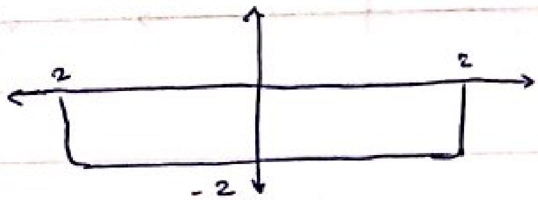
① Amp Scale $\xrightarrow{-2g(t)}$ ② Time Scale $\xrightarrow{-2g(t/4)}$ ③ Time Shift $\xrightarrow{-2g\left(\frac{t+2}{4}\right)}$

Rule: $g(t) \longrightarrow Ag\left(\frac{t-t_0}{a}\right)$

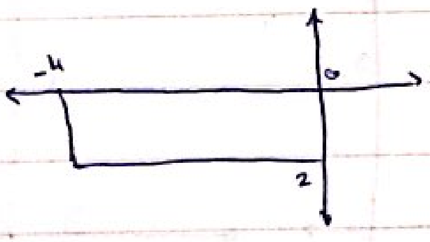
①



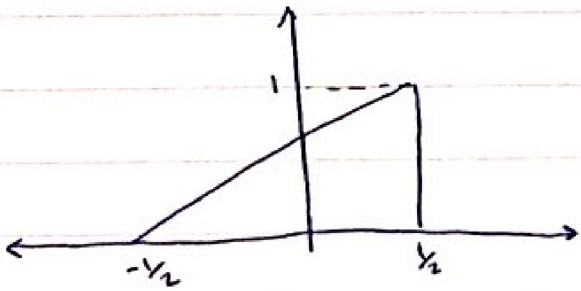
②



③



Ex: Given:



Find: $3g(-2t-1)$

$= 3g\left[-2\left(t+\frac{1}{2}\right)\right]$

$= 3g\left[\frac{t+\frac{1}{2}}{-\frac{1}{2}}\right]$

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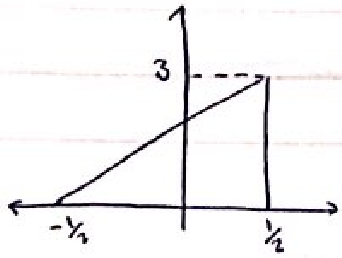
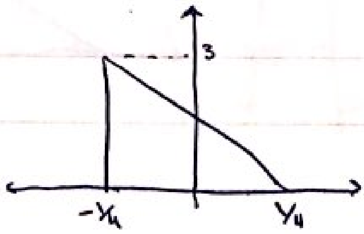
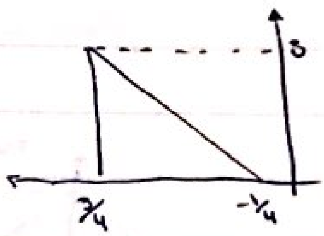
①



②

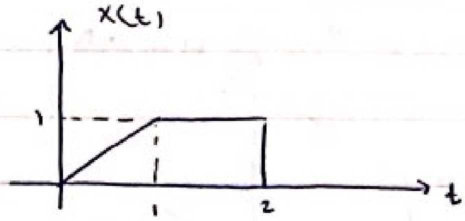


③



Given:

$$x(t) = \begin{cases} t & 0 < t < 1 \\ 1 & 1 < t < 2 \\ 0 & t > 2 \end{cases}$$

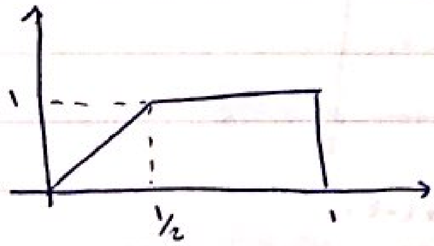


Find: $x(2t-1)$

$$= x\left[2\left(t-\tfrac{1}{2}\right)\right]$$

$$= x\left[\frac{\left(t-\frac{1}{2}\right)}{\frac{1}{2}}\right]$$

①



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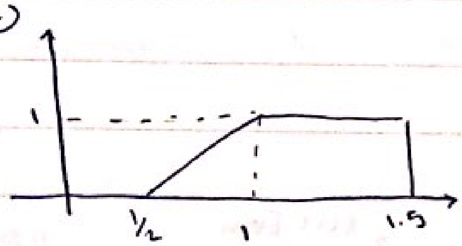
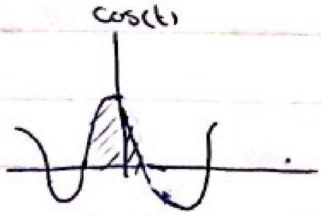


Odd / Even Function :

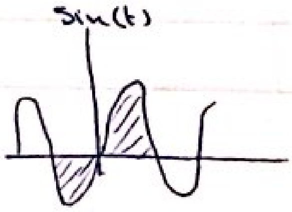
$$x(t) = x_{odd}(t) + x_{even}(t)$$

$$x_{odd}(t) = \frac{1}{2}\left[x(t) - x(-t)\right]$$

$$x_{Even}(t) = \frac{1}{2}\left[x(t) + x(-t)\right]$$

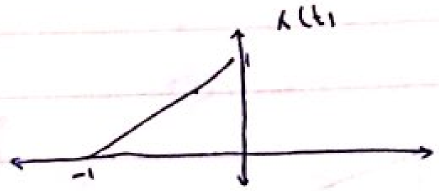


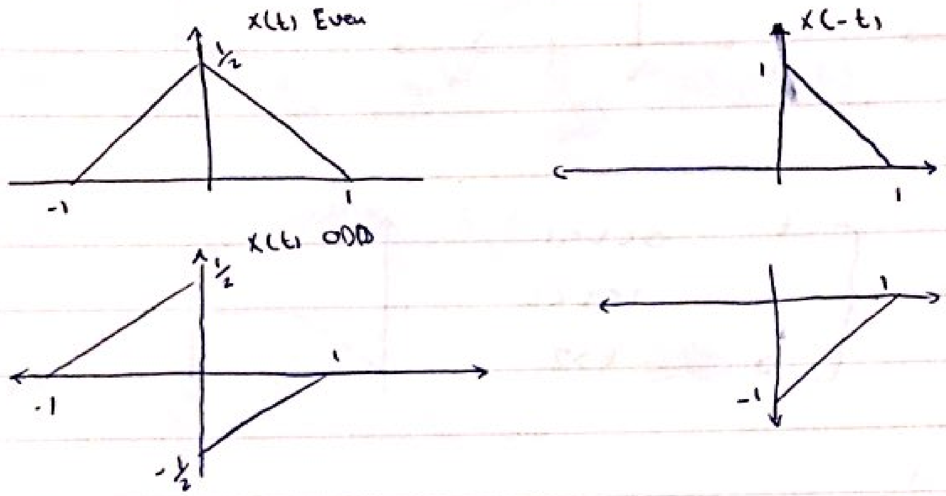
$x(t) = x(-t)$



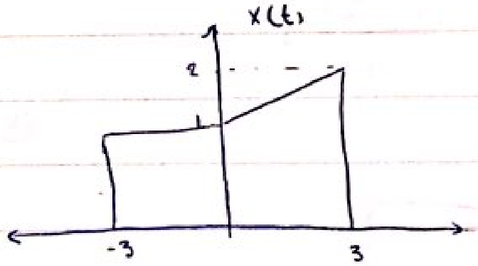
$x(t) = -x(-t)$

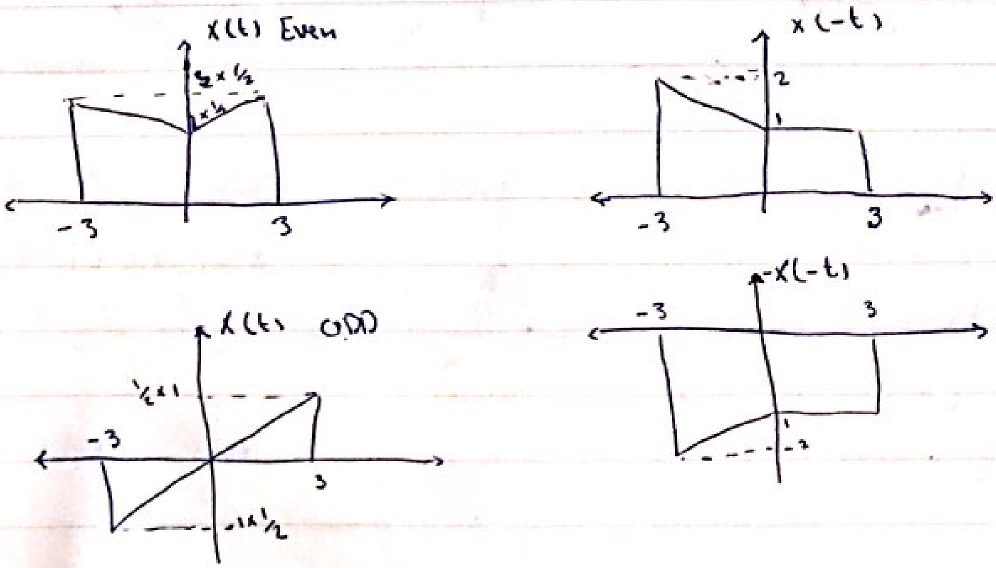
Ex: Find odd / Even parts for,



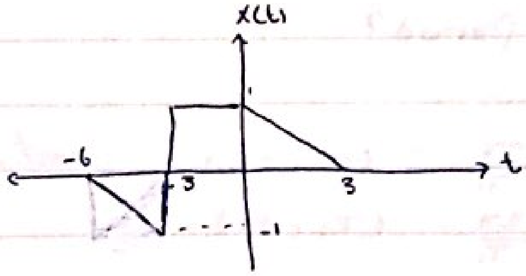


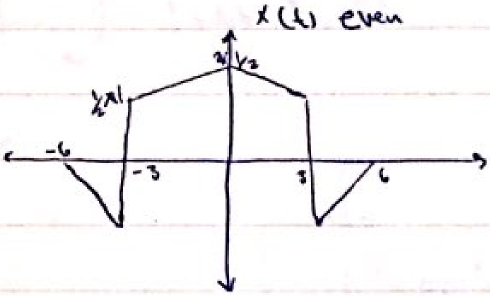
Find odd and Even

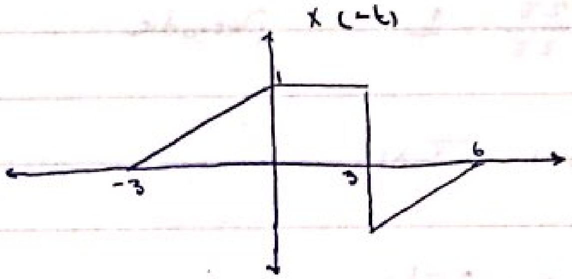


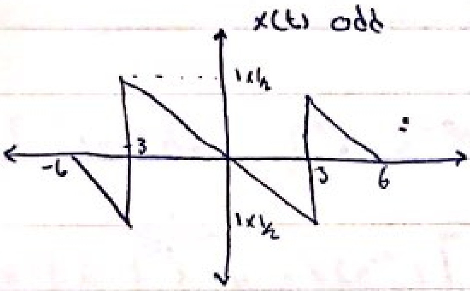


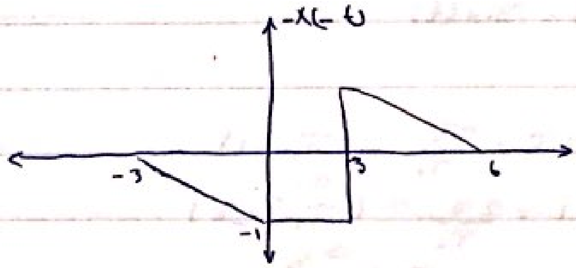
Ex:











Signal Periods :-

Rational Numbers $\longrightarrow$ 3 / 2.3 / 3.11

Irrational Numbers $\longrightarrow$ $\underset{3.14}{\pi}$ / 2.11111....

Ex: $x_1(t) = \sin(t)$

$x_2(t) = \cos(t)$

is $x(t) = x_1(t) + x_2(t)$ Periodic?

Find it's period?

$W_1 = 1 = \frac{2\pi}{T_1}$ $(T_1 = 2\pi)$

$W_2 = 1 = \frac{2\pi}{T_2}$ $(T_2 = 2\pi)$

$\frac{T_2}{T_1} = \frac{2\pi}{2\pi} = 1$ Periodic

$T_0 = T_2 \times 1 = T_1 \times 1$

Ex : $X_1(t) = \sin(2\pi t)$

$X_2(t) = \sin(t)$

$W_1 = 2\pi = \frac{2\pi}{T_1}$ $(T_1 = 1)$

$W_2' = 1 = \frac{2\pi}{T_2}$ $(T_2 = 2\pi)$

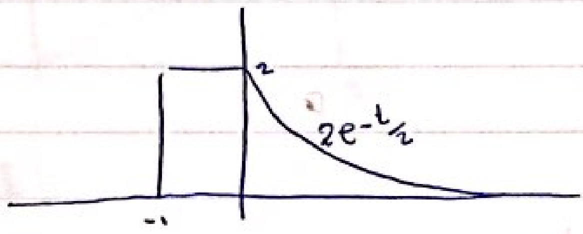
$\frac{T_2}{T_1} = \frac{2\pi}{1} = 2\pi$ non Periodic or Aperiodic

Energy:

Energy signals are limited in time
$P = 0$

$$E = \int_{-\infty}^{+\infty} |x(t)|^2 \cdot dt$$

Ex:



$$\int_{-1}^{0} |2|^2 dt + \int_{0}^{\infty} |2e^{-t/2}|^2 dt$$

$$= 4 \left(t \Big|_{-1}^{0} \right) + -4 \left(e^{-t} \Big|_{0}^{\infty} \right)$$

$e^{-\infty} = 0$
$e^{+\infty} = \infty$

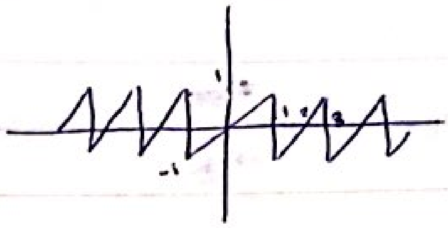
$$= -4 \left[\cancel{e}^{0} - \cancel{e}^{1} \right] = 4$$

Power:

Periodic signals are power signals
$E = \infty$

$$P = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \cdot dt$$

Ex:



$T = 2$

$P = \frac{1}{2} \int_{-1}^{1} |t|^2 . dt$

$= \int_{0}^{1} (t)^2 + \int_{1}^{2} (t-1)^2$

$\text{or} = \frac{1}{2} \left(\frac{t^3}{3} \Big|_{-1}^{+1} \right) = \frac{1}{3}$

Ex: $x(t) = A + B\cos(\omega t)$

$P_{av} = (A)^2 + \frac{(B^2)}{2}$

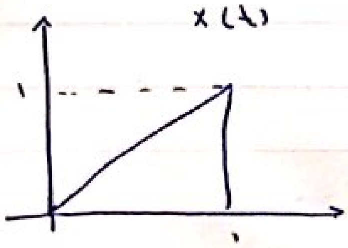
DC Power $= A^2$ | AC Power $\frac{B^2}{2}$

RMS power $= \sqrt{P_{AC}}$

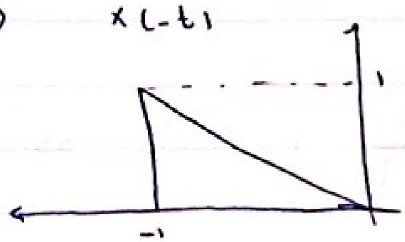
* Find a suitable measure for $x(t)$ * Q!

Ex:

(1) $x(t)$



(2) $x(-t)$



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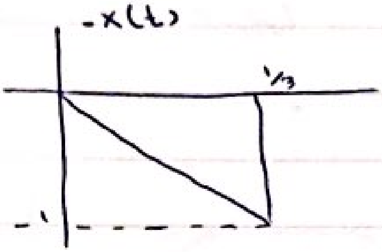

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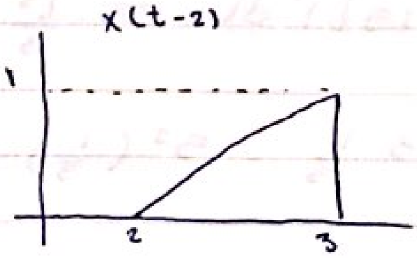

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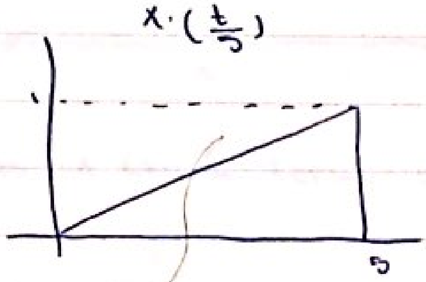
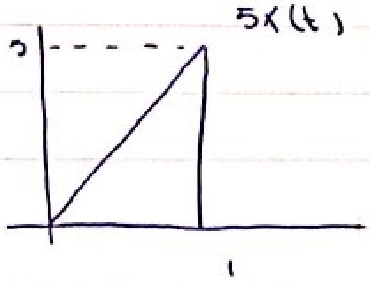


(6)



find E.

(1) $E = \int_0^1 |t|^2 \cdot dt = \frac{t^3}{3}\Big|_0^1 = \frac{1}{3}$

$= \int_{-1}^{0} |-t|^2 \cdot dt = \frac{t^3}{3}\Big|_{-1}^{0}$

$= \frac{0-(-1)}{3} = \frac{1}{3}$

*Same answer in (1),(2),(3),(4)

*(5) $E = \int_0^5 |\frac{1}{5} t|^2 \cdot dt =$

$= \frac{1}{25}\left(\frac{t^3}{3}\right)\Big|_0^5$

$= \frac{1}{25}\left(\frac{125-0}{3}\right) = \frac{5}{3} = 5\left(\frac{1}{3}\right)$

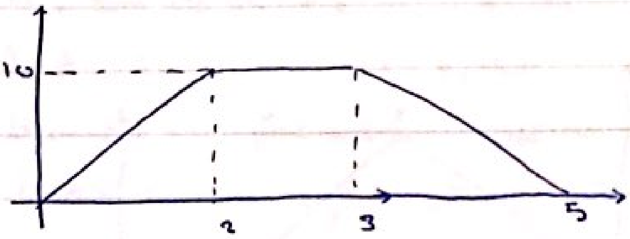
Date No

*⑥ $\int_0^1 |5t|^2 \, dt = 25 \cdot \frac{t^3}{3} \Big|_0^1$

$= 25 \cdot \frac{1}{3} = 5^2 \left(\frac{1}{3}\right)$

Qs of old Tests!

① 3-12-2015 , Test I

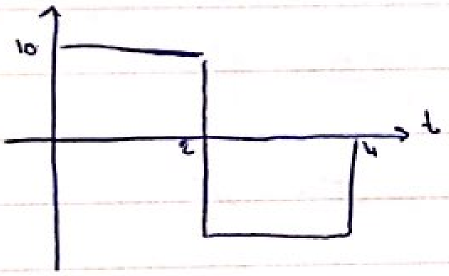


Express $x(t)$ in terms of Ramp signals.

$$x(t) = \overset{10/2}{5}\,r(t) - 5r(t-2) - 5r(t-3) + 5r(t-5)$$

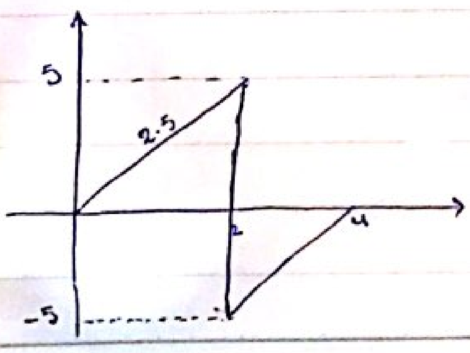
Sketch.

$$x(t) = 10u(t) - 20u(t-2) + 10u(t-4)$$



② $X(t) = 2.5r(t) - 2.5r(t-4) - 10u(t-2)$

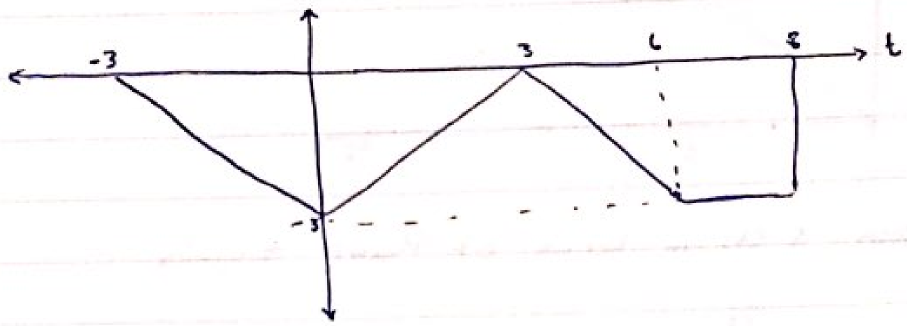
$= 2.5r(t) - 10u(t-2) - 2.5r(t-4)$



Date No

Plot: $x(t) = 2r(t) - 2r(t-3) + r(t-6) - r(t+3) + 3u(t-8)$

$$= -r(t+3) + 2r(t) - 2r(t-3) + r(t-6) + 3u(t-8)$$



Anas Alloush "4"

Date No

17-4-2014

$x_1(t) = \sin(2\pi t) + 2$, $x_2(t) = 1 - 2\sin(2\pi t)$

① find $x_3(t) = x_1(t) \cdot x_2(t)$ $\quad * \; 2\sin^2\theta = [1-\cos 2\theta]$

② Is $x_3(t)$ periodic? find it's period?

③ Find a proper signal sizing of $x_3(t)$?

① $x_3(t) = \sin(2\pi t) - 2\sin^2(2\pi t) + 2 - 4\sin(2\pi t)$

$= \sin(2\pi t) - [1-\cos[2(2\pi t)]] + 2 - 4\sin(2\pi t)$

$$x_3(t) = -3\sin\underbrace{(2\pi t)}_{f_1} + \overset{Dc}{1} + 1\cos\underbrace{(4\pi t)}_{f_2}$$

~~$x_3(t) = $~~ $w_1 = \frac{2\pi}{T_1} = 2\pi = T_1 = 1$, $w_2 = \frac{2\pi}{T_2} = 4\pi = T_2 = 0.5$

$$\frac{T_2}{T_1} \cancel{= \frac{0.5}{1}} = \frac{1}{2}$$

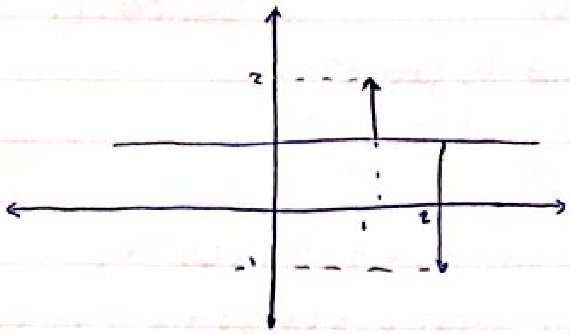
$$T_0 = \underset{1}{2T_2} = \underset{1}{T_1(1)} = 1$$

② Yes, $T_0 = 1$.

③ $P = \frac{-3^2}{2} + 1^2 + \frac{1^2}{2}$

Sketch.

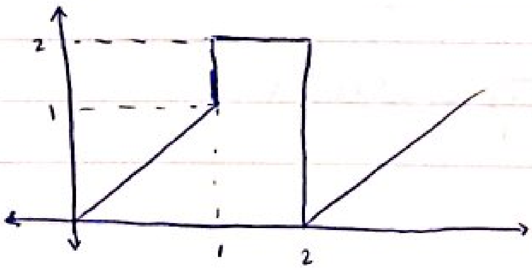
① $x(t) = 1 + \delta(t-1) - 2\delta(t-2)$



② $y(t) = \int_0^t x(t) \cdot dt$

$y(t) = \int_0^t 1 \, dt + \int_0^t \delta(t-1) + \int_0^t -2\delta(t-2)$

$= t\, \underline{u(t)} + u(t-1) - 2u(t-2)$



$\int_{-5}^{-2} (x t^2 + t) \cos(2\pi t) \cdot \delta(t + \overset{-(-4)}{4}) \, dt = ?$

$= (16x - 4) \cdot \cos(-8 \times 180)$

$\int_{-10}^{10} \cos(2\pi t) \cdot \delta(5t+3)$

$= \frac{1}{5} \cos(2\pi \cdot \frac{-3}{5})$

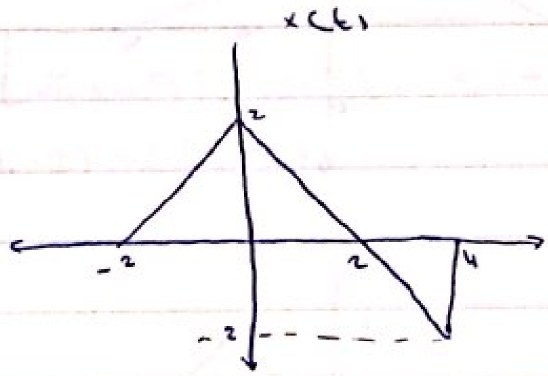
Date No

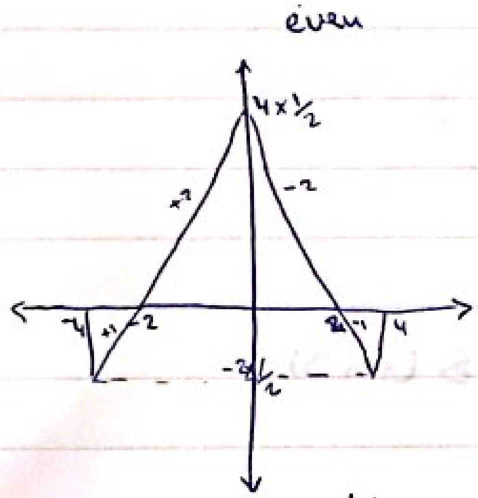
$$V(t) = \begin{cases} 2+t & -2 \le t < 0 \\ 2-t & 0 < t < 4 \\ 0 & e.w \end{cases}$$

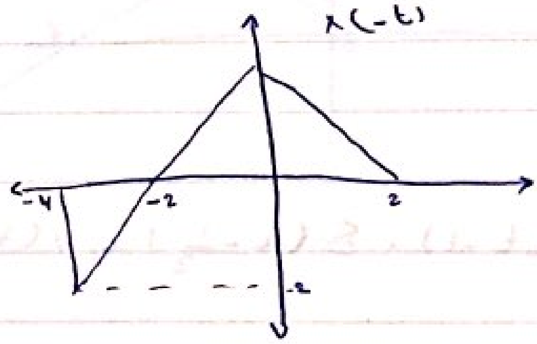
① Sketch V(t) odd / even

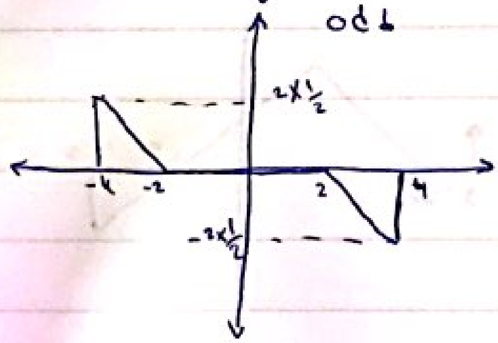
$x(t) = 2-t$

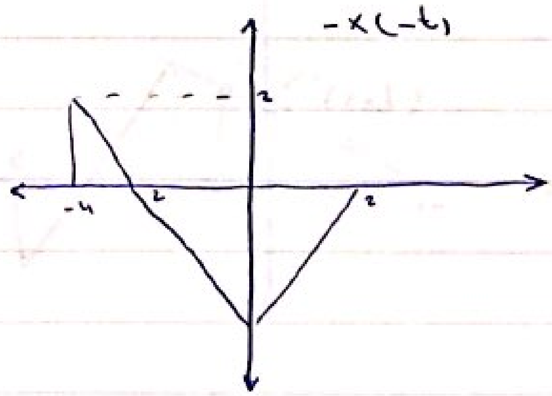
$(-2) = 2-4$





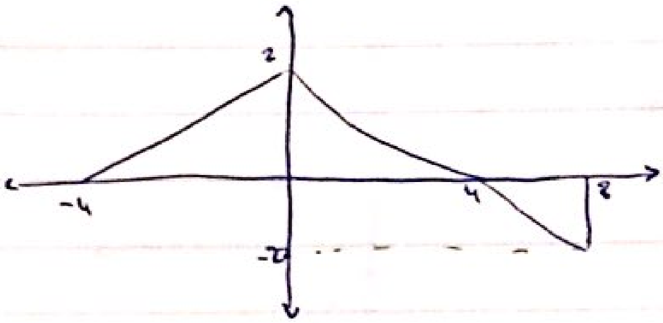


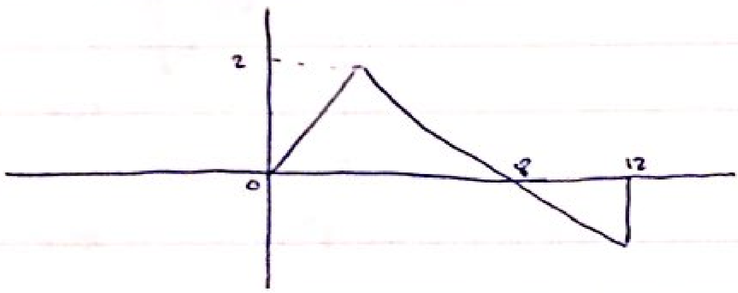




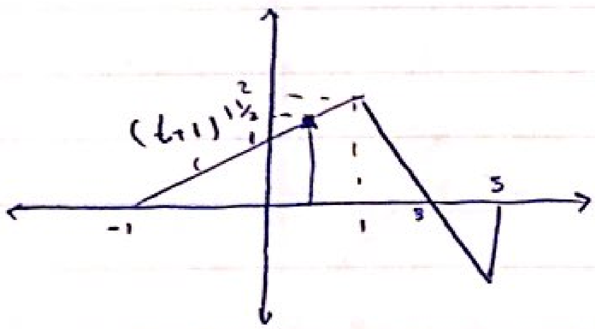
Date No

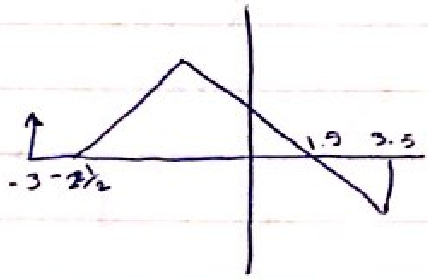
Sketch (2) $v[0.5(t-4)]$

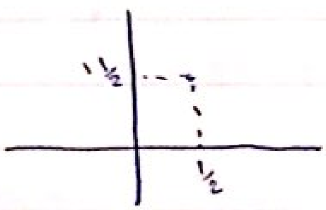


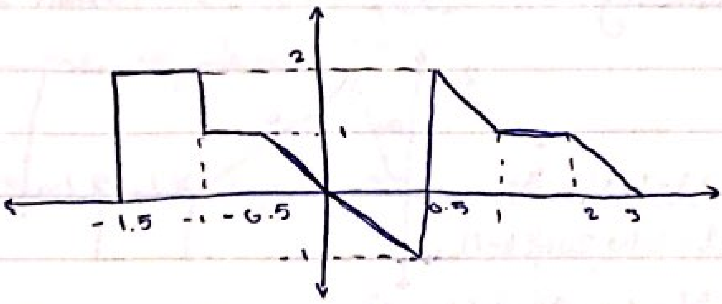


(3) $v(t-1) \cdot \delta(t-\frac{1}{2}) + v(t+\frac{1}{2})\,\delta(t+3)$



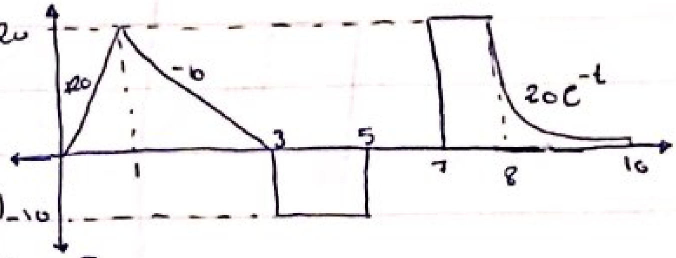






$$x(t) = 2u(t+1.5) - u(t+1) - 2r(t+0.5) + {}^{*}\Big[2r(t-0.5) + 3u(t-0.5) - 2r(t-0.5)\Big] + 2r(t-1) - r(t-2) + r(t-3)$$

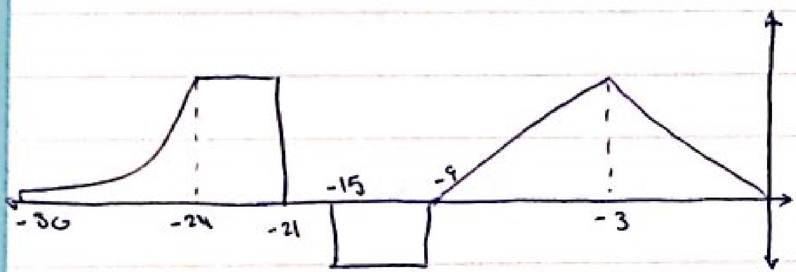
① Express f(t) Mathematically Final 2/3/2013

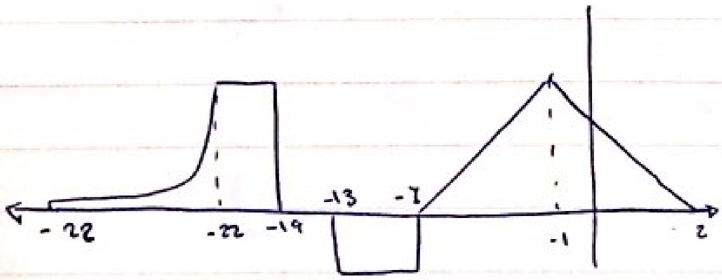


$f(t) = 20r(t) - 30r(t-1) + 10r(t-3)$
$- 10r(t-5) + 10u(t-5) + 20u(t-7)$
$\underbrace{-20u(t-8)}_{①} + \underbrace{20e^{-t}[u(t-8) - u(t-10)]}_{②}$

② sketch $f\left(\frac{2-t}{3}\right) \longrightarrow Ag\left(\frac{t-t_0}{a}\right)$

$f\left(\frac{-(t-2)}{3}\right)$

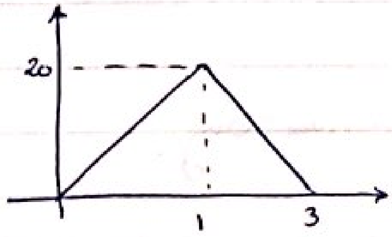




Date No

③ sketch $f(t) \cdot u(3-t) = y(t)$ and write the equation.

$y(t) = 20r(t) - 30r(t-1) + 10r(t-3)$



④ sketch $\int_1^3 f(t) \cdot \delta(2t - 4)\, dt = \frac{1}{|a|} f\left(\frac{b}{a}\right)$
and write the equation.

$\frac{1}{2} f\left(\frac{4}{2}\right)$ $y = mx + c$

$0 = -10 \times 3 + c$

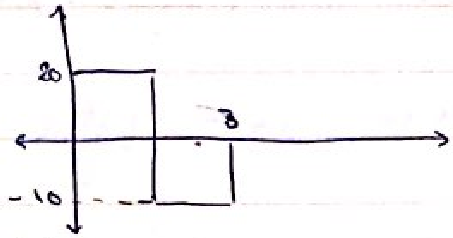
$c = 30$

$= \frac{1}{2} \times f(t) = -10t + 30$

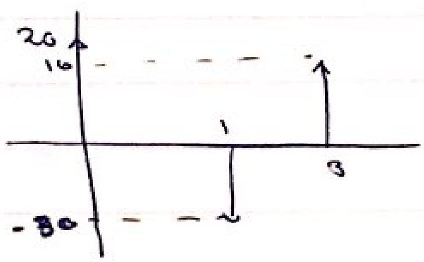
$= \frac{1}{2}(-10(2) + 30) = 5$

⑤ sketch and write equation $y(t) = \frac{dy(t)}{dt}$

$\frac{d}{dt} y(t) = 20u(t) - 30u(t-1) + 10u(t-3)$



$\frac{dy}{dt} = 20\delta(t) - 30\delta(t-1) + 10\delta(t-3)$



$x_1(t) = 2\cos^2(\frac{\pi}{3}t - \pi)$

$x_2(t) = u(t-1.5) - u(t-3) + u(t-6) - u(t-9)$

① find $x_1(t)$ time delay

② sketch $x_3(t) = x_1(t) \cdot x_2(t)$

③ Is $x_3(t)$ periodic?

Solution:

Recall $\longrightarrow \cos 2\theta = 2\cos^2\theta - 1$ $\qquad \cos\theta = \sin(\theta + \frac{\pi}{2})$

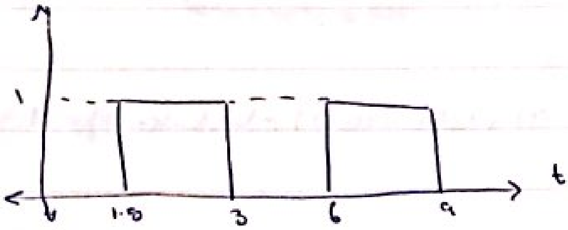
$\cos 2\theta + 1 = 2\cos^2\theta$

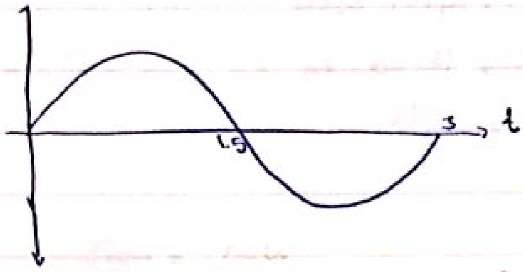
$\therefore x_1(t) = 1 + \cos[\frac{2\pi}{3}t - 2\pi]$

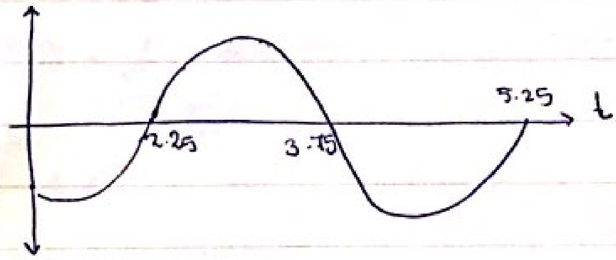
$\therefore x_1(t) = 1 + \sin[\frac{2\pi}{3}t - 2\pi + \frac{\pi}{2}]$

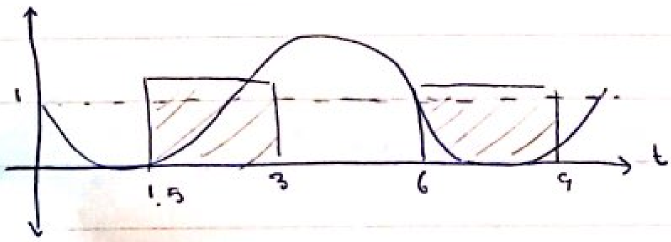
$x_1(t) = 1 + \sin[\frac{2\pi}{3}t - 1.5\pi]$

Time delay $t_d = \frac{-\phi}{\omega_0} = \frac{-(-1.5\pi)}{2\pi/3} = 2.25$









Not Periodic.

$$E = \int_{1.5}^{3} 1 + \sin(\dots)^2 + \int_{6}^{9} 1 + \sin(\dots)^2$$

Anas Alloush «6»

Date No

20-11-2014

Q1. $x_1(t) = \sin(\pi t/_3 - \pi)$

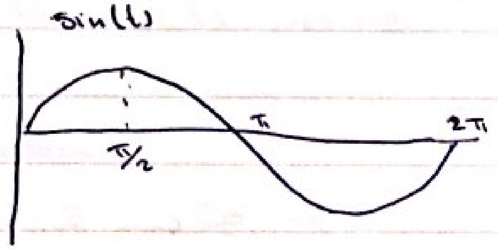
$x_2(t) = u(t-3) - u(t-9) + u(t-10.5) - u(t-10.5) - u(t-13.5)$

① find t_0 of $x_1(t)$

② Find and sketch $x_3(t) = x_1(t) . x_2(t)$

③ is $x_3(t)$ Periodic? find it's period

④ Find a suitable measure of $x_3(t)$

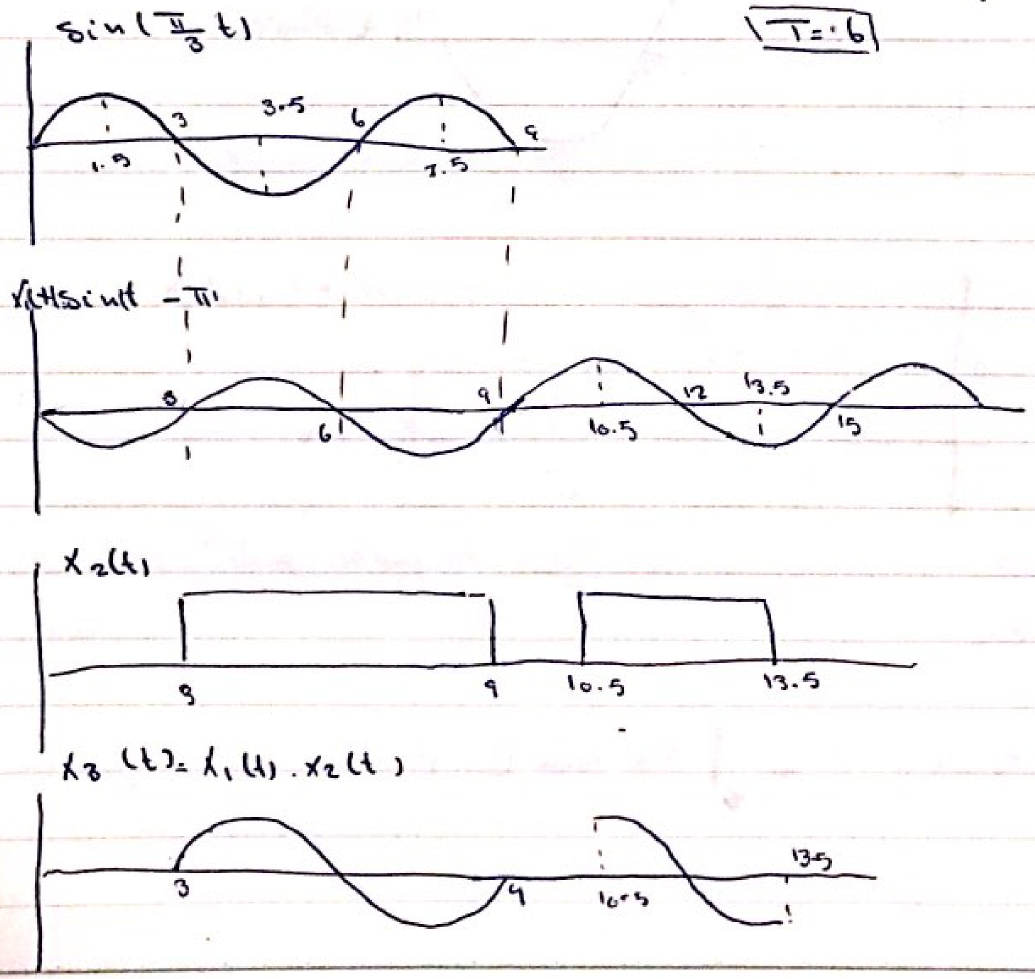


$W = 1 = \frac{2\pi}{T}$

$T = 2\pi$

$W = \frac{\pi}{3} = \frac{2\pi}{T}$

$\boxed{T = 6}$



Date No

$$t_d = \frac{-\phi}{\omega_0} \qquad \sin(\omega t + \phi)$$

$$t_d = \frac{-(-\pi)}{\pi/3} = 3$$

$$E_{total} = E_1 + E_2$$

$$= \int_3^9 \left|\sin\left(\frac{\pi}{3}t - \pi\right)\right|^2 dt + \int_{6.5}^{13.5} \left|\sin\left(\frac{\pi}{3}t - \pi\right)\right|^2 dt$$

$$x(t) = Ae^{bt} \qquad b>0$$

$$E = \int_{-\infty}^{+\infty} |Ae^{bt}|^2 = \int_{-\infty}^{+\infty} A^2 e^{2bt} = A^2\left(\frac{e^{2bt}}{2b}\right)_{-\infty}^{+\infty} = \boxed{\infty}$$

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{+T/2} |Ae^{bt}|^2 = \lim_{T\to\infty} \frac{1}{T} \left[\frac{A^2 e^{2bt}}{2b}\right]_{-T/2}^{T/2} = \boxed{0}$$

$x(t)$ neither Power nor Energy

$$x(t) = Ae^{j\pi t}$$

$$e^{j\phi} = \cos\phi + \cancel{j\sin\phi}$$

$$= A\cos\pi t$$

$$= \frac{A^2}{2}$$

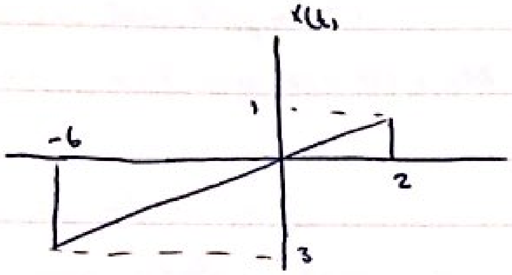
Scanned by CamScanner

Date No

3-12-2015

Test # 1

Q. -



$x_{even} = \frac{1}{2}(x(t) + x(-t))$

$x_{odd} = \frac{1}{2}(x(t) - x(-t))$

$y = mx + c$

$x(t) = \frac{1}{2}t + c$

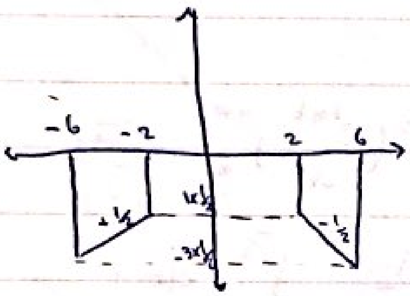
$x(t) = \frac{1}{2} - 2 = -1$

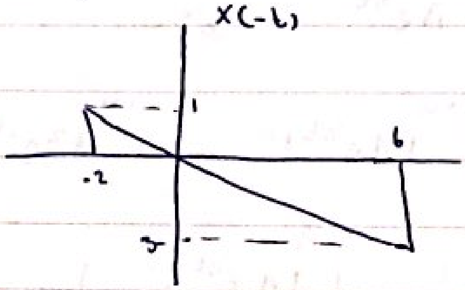
① find odd / Even

② Sketch $x(9-3t)$

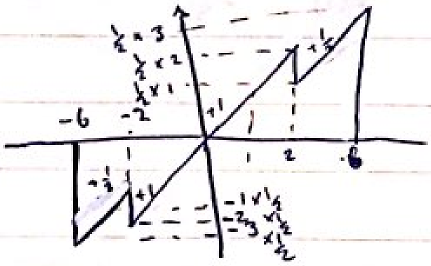
③ find suitable measure for $3x\left(\frac{t-1.5}{4}\right)$

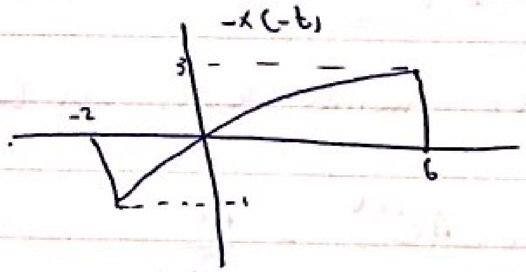
Even





ODD





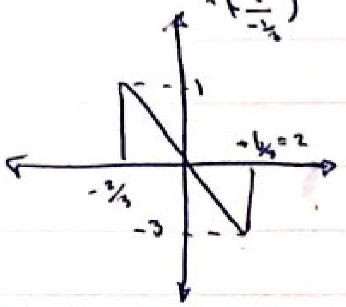
$Ag[(\frac{t-t_0}{a})]$

$x(9-3t) = x[-3(t-3)]$

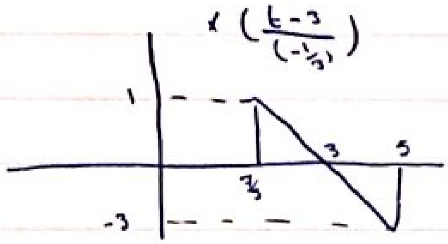
$= x\left[\frac{[t-3]}{-\frac{1}{3}}\right]$

① Time scalling $x\left(\frac{t}{-\frac{1}{3}}\right)$

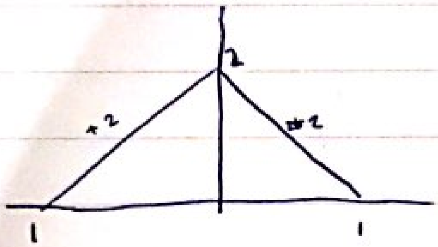
Scanned by CamScanner



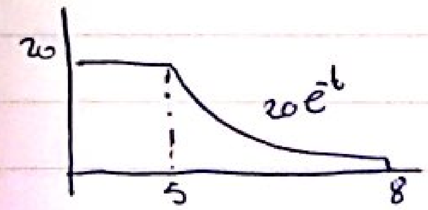
② Time shifting



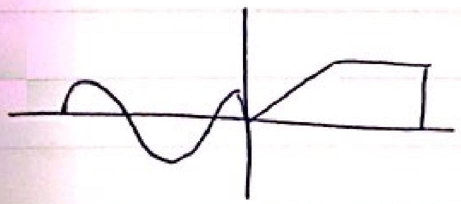
$E = \int_{-6}^{2} \left| \frac{1}{2} t \right|^2 dt = [\,] \times 3^2 \times 4$



$2r(t+1) - 4r(t) + 2r(t-1)$



$20u(t) - 20u(t-5) + 20e^{-t}[u(t-5) - u(t-8)]$



$x(t) = \sin(wt)[u(-t)] + r(t) + \ldots$